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9709/31

May/June 2024

1 hour 50 minutes

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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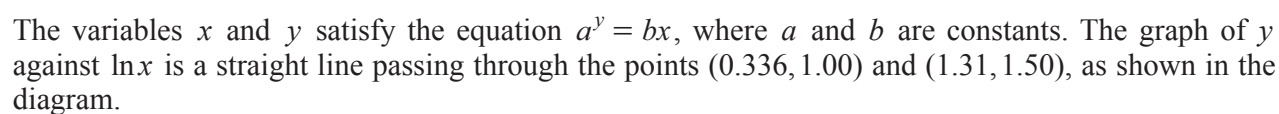
[Turn over

- 1 Expand $(3+x)(1-2x)^{\frac{1}{2}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

[illegible]

- 2 Solve the equation $\ln(x-5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

4 The complex number u is given by $u = -1 - i\sqrt{3}$.

- (a) Express u in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ . [2]

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The complex number v is given by $v = 5\left(\cos \frac{1}{6}\pi + i \sin \frac{1}{6}\pi\right)$.

- (b) Express the complex number $\frac{v}{u}$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

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- 5** The equation of a curve is $y = \frac{e^{\sin x}}{\cos^2 x}$ for $0 \leq x \leq 2\pi$.

Find $\frac{dy}{dx}$ and hence find the x -coordinates of the stationary points of the curve. [7]

[illegible]

- 6 (a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} \frac{1}{2}x = e^x - 3$ has exactly one root, denoted by α , in the interval $0 < x < \pi$. [2]

- (b)** Verify by calculation that α lies between 1 and 2. [2]

- (c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula

$$x_{n+1} = \ln(\operatorname{cosec} \frac{1}{2}x_n + 3)$$

converges, then it converges to α .

[1]

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- (d) Use this iterative formula with an initial value of 1.4 to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (e) State the minimum number of calculated iterations needed with this initial value to determine α correct to 2 decimal places. [1]

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- 7 (a) On a single Argand diagram sketch the loci given by the equations $|z - 3 + 2i| = 2$ and $|w - 3 + 2i| = |w + 3 - 4i|$ where z and w are complex numbers. [4]

- (b)** Hence find the least value of $|z - w|$ for points on these loci. Give your answer in an exact form. [2]

[illegible]

Give your answer in the form $a+b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

[illegible]

- 9 The equations of two straight lines l_1 and l_2 are

$$l_1: \mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad l_2: \mathbf{r} = -\mathbf{i} - \mathbf{j} - \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}),$$

where a is a constant.

The lines l_1 and l_2 are perpendicular.

- (a) Show that $a = 4$. [1]

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The lines l_1 and l_2 also intersect.

- (b) Find the position vector of the point of intersection. [4]

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The point A has position vector $-5\mathbf{i} + \mathbf{j} - 9\mathbf{k}$.

- (c) Show that A lies on l_1 . [2]

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The point B is the image of A after a reflection in the line l_2 .

- (d) Find the position vector of B . [2]

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10 (a) Given that $2x = \tan y$, show that $\frac{dy}{dx} = \frac{2}{1+4x^2}$. [3]

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(b) Hence find the exact value of $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x \tan^{-1}(2x) dx$. [7]

This image shows a blank sheet of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the paper.

- 11 In a field there are 300 plants of a certain species, all of which can be infected by a particular disease. At time t after the first plant is infected there are x infected plants. The rate of change of x is proportional to the product of the number of plants infected and the number of plants that are **not** yet infected. The variables x and t are treated as continuous, and it is given that $\frac{dx}{dt} = 0.2$ and $x = 1$ when $t = 0$.

(a) Show that x and t satisfy the differential equation

$$1495 \frac{dx}{dt} = x(300 - x). \quad [2]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Using partial fractions, solve the differential equation and obtain an expression for t in terms of a single logarithm involving x . [9]

[illegible]

[illegible]

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